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SMOOTHNESS OF SOLUTIONS OF STOCHASTIC EVOLUTION EQUATIONS AND THE EXISTENCE OF A FILTERING TRANSITION DENSITY

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In this paper, we shall discuss the smoothness of solutions of stochastic evolution equations, which has been investigated in N. V. Krylov and B. L. Rozovskii [2] [3], to establish the existence of a filtering transition density.

First, we introduce the filtering equation, which has been discussed in [1] [3] [6] and [9]. Let us consider the system (x_t, y_t) given by the stochastic differential equation

$$\begin{aligned} dx_t &= a(x_t, y_t, t)dt + b(x_t, y_t, t)d\nu_t \\ dy_t &= A(x_t, y_t, t)dt + B(y_t, t)d\nu_t \\ x_0 &= \theta,\ y_0 = \xi,\ t \in [0, T],\ T < +\infty\ , \end{aligned}$$

where $\nu = \{\nu_t\}_{t\in[0,T]}$ is a $(d + d_1)$-dimensional Brownian motion defined on a complete probability space, and a, A, b and B are matrices of type $d \times 1$, $d_1 \times 1$, $d \times (d + d_1)$ and $d_1 \times (d + d_1)$ respectively. We denote by F_t^Y the complete σ-algebra $\sigma\{y_\tau, 0 \leqq \tau \leqq t\}$. Let us denote by $P_t[f]$ a measurable modification of the conditional expectation $E[f(x_t, y_t, t)\,|\,F_t^Y]$. We put

$$\begin{aligned} C &= (BB^*)^{-1/2}\ , \qquad \beta(x, y, t) = CA\ , \\ \overline{w}_t &= \int_0^t C(y_\tau, \tau)dy_\tau - \int_0^t P_\tau[\beta]d\tau\ , \\ y'_t &= \overline{w}_t + \int_0^t P_\tau[\beta]d\tau \end{aligned}$$

and

$$\rho_t = \exp\left\{-\int_0^t P_\tau[\beta]d\overline{w}_\tau - \frac{1}{2}\int_0^t |P_\tau[\beta]|^2\, d\tau\right\}\ .$$

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