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SUPER-ŁUKASIEWICZ PROPOSITIONAL LOGICS

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§ 0. Introduction

In [8] (1920), Łukasiewicz introduced a 3-valued propositional calculus with one designated truth-value and later in [9], Łukasiewicz and Tarski generalized it to an m-valued propositional calculus (where m is a natural number or $\aleph_0$) with one designated truth-value. For the original 3-valued propositional calculus, an axiomatization was given by Wajsberg [16] (1931). In a case of $m \neq \aleph_0$, Rosser and Turquette gave an axiomatization of the m-valued propositional calculus with an arbitrary number of designated truth-values in [13] (1945). In [9], Łukasiewicz conjectured that the $\aleph_0$-valued propositional calculus is axiomatizable by a system with modus ponens and substitution as inference rules and the following five axioms: $p \supset q \supset p$, $(p \supset q) \supset (q \supset r) \supset p \supset r$, $p \vee q \supset q \vee p$, $(p \supset q) \vee (q \supset p)$, $(\sim p \supset \sim q) \supset q \supset p$. Here we use $P \vee Q$ as the abbreviation of $(P \supset Q) \supset Q$. We associate to the right and use the convention that $\supset$ binds less strongly than $\vee$. In [15] p. 51, it is stated as follows: "This conjecture has proved to be correct; see Wajsberg [17] (1935) p. 240. As far as we know, however, Wajsberg's proof has not appeared in print." Rose and Rosser gave the first proof of it in print in [12] (1958). Their proof was essentially due to McNaughton's theorem [10], so it was metamathematical in nature. An algebraic proof was given by Chang [1] [2] (1959).

On the other hand, Rose [11] (1953) showed that the cardinality of the set of all super-Łukasiewicz propositional logics is $\aleph_0$. Surprisingly it was before Rose and Rosser's completeness theorem [12]. The proof in Rose [11] was also due to McNaughton's theorem. Some of our theorems in this paper have already been obtained by Rose [11]. But our proofs are completely algebraic.

In our former paper [5], we gave a complete description of super-

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