QUASILINEAR WAVE EQUATIONS AND RELATED NONLINEAR EVOLUTION EQUATIONS*

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Introduction

In this paper we consider the relations between quasilinear wave equations

$$(1) \qquad u_{tt} - \sum_{i,j=1}^{n} a_{ij}(\mathrm{grad}\, u)u_{ij} + cu_t = f,$$

and related third-order differential equations

$$(2) \qquad u_{tt} - \lambda \Delta u_t - \sum_{i,j=1}^{n} a_{ij}(\mathrm{grad}\, u)u_{ij} + cu_t = f,$$

(λ is a positive parameter) with the same initial conditions

$$(3) \qquad \begin{cases} u(x, 0) = u_0(x), \\ u_t(x, 0) = v_0(x), \end{cases}$$

where $x = (x_1, x_2, \cdots, x_n)$, $t \geq 0$, $u = u(x, t)$, $u_t = \partial u/\partial t$, $u_{tt} = \partial^2 u/\partial t^2$, $u_i = \partial u/\partial x_i$, $u_{ij} = \partial^2 u/\partial x_i \partial x_j$, $\mathrm{grad}\, u = (u_1, u_2, \cdots, u_n)$ and $\Delta u = \sum_{i=1}^{n} u_{ii}$. In equations (1) and (2), c is a real number and $a_{ij} = a_{ji}$ ($i, j = 1, 2, \cdots, n$) are real smooth functions satisfying

$$\sum_{i,j=1}^{n} a_{ij}(\eta)\xi_i\xi_j \geq a_0(\rho)|\xi|^2, \qquad |\xi|^2 = \sum_{i=1}^{n} \xi_i^2,$$

for every $\eta \in R^n$, $|\eta| \leq \rho$ and $\xi \in R^n$, $a_0(\rho)$ being a positive non-increasing function.

Equations of type (2) were first proposed by Greenberg, MacCamy and Mizel [7] in the case $n = 1$. They have dealt with equations of the form

$$(4) \qquad u_{tt} - \sigma(u_x)u_{xx} - \lambda u_{xtx} = 0, \qquad \lambda > 0,$$

to approach quasilinear wave equations for a nonlinear string model

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