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SOME REMARKS CONCERNING DEMAZURE'S CONSTRUCTION OF NORMAL GRADED RINGS

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Introduction

In [1], Demazure showed a new way of constructing normal graded rings using the concept of "rational coefficient Weil divisors" of normal projective varieties and he showed, among other things, the following

THEOREM ([1], 3.5). *If* $R = \oplus_{n\geq 0} R_n$ *is a normal graded ring of finite type over a field* k *and if* T *is a homogeneous element of degree* 1 *in the quotient field of* R, *then there exists unique divisor* $D \in \mathrm{Div}(X, \boldsymbol{Q})$ $(X = \mathrm{Proj}(R))$, *such that* $R_n = H^0(X, \mathcal{O}_X(nD))\cdot T^n$ *for every* $n \geqq 0$. *(See* (1.1) *for the definition of* $\mathrm{Div}(X, \boldsymbol{Q})$ *and* $\mathcal{O}_X(nD)$.*)*

Let us denote the ring R above by $R = R(X, D)$. In this note we want to consider the following problems concerning $R = R(X, D)$.

(1) What is the depth of R? In particular, when is R a Macaulay ring or a Gorenstein ring?

(2) When is R a rational singularity?

The paper is divided into three sections. In § 1, we calculate the divisor class group of R. Although the contents of this section are included implicitly in [1], we need to state the results explicitly to define the canonical class $\mathrm{cl}(K_R)$ of R in § 2.

In § 2, we seek the condition for R to be a Macaulay ring or a Gorenstein ring. First, we express the local cohomology groups of R by the cohomology groups of $\mathcal{O}_X(nD)$ $(n \in \boldsymbol{Z})$. Then, using Grothendieck duality, we calculate the canonical class $\mathrm{cl}(K_R)$ of R and, in particular, we can find the condition for R to be a Gorenstein ring.

In § 3, we establish a criterion for R to be a rational singularity when X is smooth and $\mathrm{Supp}(D - \lfloor D \rfloor)$ has only normal crossings as its singularity. (See (1.1) for the definition of $\lfloor D \rfloor$.) This criterion gives us very

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