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WHEN IS A REGULAR SEQUENCE SUPER REGULAR?

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Let $(B, \mathscr{F})$ be a filtered, noetherian ring. A sequence $x = x_1, \cdots, x_n$ in B is called super regular if the sequence of initial forms

$$\xi_1 = L(x_1), \cdots, \xi_n = L(x_n)$$

is a regular sequence in $gr_{\mathscr{F}}(B)$.

If B is local and the filtration $\mathscr{F}$ is $\mathfrak{A}$-adic then any super regular sequence is also regular, see [6], 2.4.

In [3], Prop. 6 Hironaka shows that in a local ring $(B, \mathfrak{M})$ an element $x \in \mathfrak{M} \backslash \mathfrak{M}^2$ is super regular (with respect to the $\mathfrak{M}$-adic filtration) if and only if x is regular in B and $(x) \cap \mathfrak{M}^{n+1} = (x)\mathfrak{M}^n$ for every integer n.

This result is extended to a more general situation in [6], 1.1. In the present paper we will characterize super regular sequences in a relative case:

Let A be a regular complete local ring, $B = A/I$ an epimorphic image of A and $x = x_1, \cdots, x_n$ a regular sequence in B which is part of a minimal system of generators of the maximal ideal of B. Let $y = y_1, \cdots, y_n$ be a sequence in A which is mapped onto x. Then y is part of a regular system of parameters of A. Therefore y is a super regular sequence in A.

We put $\bar{A} = A/(y)A$, $\bar{I} = I/(y)I$ and $\bar{B} = B/(x)B$. Then $\bar{B} = \bar{A}/\bar{I}$, since x is a B-sequence.

As a consequence of our main result, the following conditions are equivalent:

(a) x is a super regular sequence in B

(b) For all elements $g \in \bar{I}$ there exists $f \in I$, such that

$$\bar{f} = g \quad \text{and} \quad \nu(f) = \nu(g) .$$

(Here $\bar{f}$ denotes the image of f in $\bar{I}$ and $\nu(f)$ the degree of the initial form

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