S. Itatsu
Nagoya Math. J.
Vol. 83 (1981), 37-51

ERGODIC PROPERTIES OF THE EQUILIBRIUM MEASURE OF THE STEPPING STONE MODEL IN POPULATION GENETICS

SEIICHI ITATSU

§ 1. Introduction

We shall present in this paper some ergodic properties of the stepping stone model. The model has been proposed by M. Kimura [2], to describe the evolution of a genetical population with mating and geographical structures. It has been investigated and developed by M. Kimura and G. H. Weiss [3], G. H. Weiss and M. Kimura [6], W. Fleming and C. -H. Su [1], S. Sawyer [5], and others.

The model is assumed to have infinitely many colonies, which are discretely distributed, and each of which has individuals of the same number N. We also assume that migrations take place from colony to colony, that genes are subject to mutation, and that random sampling of individuals occurs within a colony. Here the random sampling means that pairs of genes are sampled from the gene pool of sufficiently large numbers.

We shall consider the spatial distributions of gene frequencies introduced on the colony space and discuss the time evolution of the distributions by using a Markov chain. S. Sawyer [5] investigated the time evolution of the stepping stone model and obtained the convergence properties of the probability of that any two individuals randomly chosen from different colonies are always genetically identical in the n-th generation. In this paper we are interested in the ergodic property of the stepping stone model which is more finer and stronger than S. Sawyer's results. In fact we shall show that under the assumption of the existence of mutation the probability measure of the distribution of the frequencies on the colony space converges to a limit measure, which is a unique

Received March 26, 1979.