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Nagoya Math. J.
Vol. 83 (1981), 15–36

HOMOTOPY CLASSIFICATION OF CONNECTED SUMS OF SPHERE BUNDLES OVER SPHERES, I

Dedicated to Professor A. Komatu on his 70th birthday

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Introduction

In the classification problems of manifolds, the connected sums of sphere bundles over spheres appear frequently. In fact, the manifolds with certain tangential and homotopy properties come to such connected sums (cf. Tamura [15], [16], Ishimoto [6], [8], [9]). Motivated by those, in this paper and the subsequent paper, we classify connected sums of sphere bundles over spheres up to homotopy equivalence by extending the results of I. M. James and J. H. C. Whitehead [10], [11], which correspond to the case that the connected sums of the above happen to be single sums. We also use Wall [17] in the case when the fibres and the base spaces of bundles are same dimensional.

In this paper*⁾, we treat with the case that bundles admit cross-sections, and in the subsequent paper, we discuss the general case.

Let A be a p-sphere bundle over a q-sphere $(p, q > 1)$ which admits a cross-section, and consider the diagram

$$\begin{tikzcd} & \pi_{q-1}(SO_p) \arrow[r, "i_*"] \arrow[d, "J"] & \pi_{q-1}(SO_{p+1}) \arrow[d, "J"] \\ \pi_q(S^p) \arrow[ur, "\partial"] \arrow[r, "P"'] & \pi_{p+q-1}(S^p) \arrow[r, "E"] & \pi_{p+q}(S^{p+1}) \end{tikzcd},$$

where ∂, i_* belong to the homotopy exact sequence of the fibering $SO_p \to SO_{p+1} \to S^p = SO_{p+1}/SO_p$, $P = [\ ,\iota_p]$ (the Whitehead product with the orientation generator ι_p of $\pi_p(S^p)$), E is the suspension homomorphism, and J means the J-homomorphism. The diagram commutes up to sign,

Received October 21, 1978.

* The following results were previously announced without detailed proofs in Proc. Japan Acad. 55 (1979), 306–308.