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REALIZATION OF CHERN CLASSES BY SUBVARIETIES WITH CERTAIN SINGULARITIES

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§ 0. Introduction

In this paper we are concerned with subvarieties which realize Chern classes of holomorphic vector bundles. The existence of these subvarieties is known in some cases (for instance, see A. Grothendieck [2] for projective algebraic varieties and M. Cornalba and P. Griffiths [1] for Stein manifolds). In the present paper we realize Chern classes by subvarieties with singularities of a certain type. Our main theorem is as follows (see Def. 1.1.3 for the definition of quasilinear subvarieties).

MAIN THEOREM. *Let M be a paracompact complex manifold of dimension n and $\xi = (E, \pi, M)$ a holomorphic vector bundle of rank q with the condition* 2.2.1. *Then, for any integer $1 \leq k \leq n$, there exists a subvariety V of M such that*

(a) *V realizes the k-th Chern class of ξ.*
(b) *V is quasilinear of degree $k-1$ and can be desingularized by means of σ-processes.*
(c) *In particular, V is non-singular for $[n/2] \leq k \leq n$.*

As an application of Main Theorem, we show in § 4.3 that Chern classes of arbitrary holomorphic vector bundles over Stein manifolds can be realized by quasilinear subvarieties.

The following is an outline of the proof of Main Theorem. Let Φ be a holomorphic map from M into the complex Grassmann manifold $G_{q,m}$ which induces the bundle. We regard Φ as a holomorphic map from M into $G_{q,p+m}$ through an embedding $G_{q,m} \subset G_{q,p+m}$. Given a holomorphic map f from the total space E into the complex euclidean space C^p, we associate to f a holomorphic map Φ_f from M into $G_{q,p+m}$. We deform Φ into Φ_f so that Φ_f is transversal to all the strata of the Schubert variety F_1 in $G_{q,p+m}$.

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