S. Shirai
Nagoya Math. J.
Vol. 75 (1979), 133-143

ON THE CENTRAL IDEAL CLASS GROUP OF CYCLOTOMIC FIELDS

SUSUMU SHIRAI

Introduction

Let $\boldsymbol{Q}$ be the rational number field, $K/\boldsymbol{Q}$ be a finite Galois extension with the Galois group G, and let C_K be the ideal class group of K in the wider sense. We consider C_K as a G-module. Denote by I the augmentation ideal of the group ring of G over the ring of rational integers. Then $C_K/I(C_K)$ is called the central ideal class group of K, which is the maximal factor group of C_K on which G acts trivially. A. Fröhlich [3, 4] rationally determined the central ideal class group of a complete[1)] Abelian field over $\boldsymbol{Q}$ whose degree is some power of a prime. The proof is based on Theorems 3 and 4 of Fröhlich [2]. D. Garbanati [6] recently gave an algorithm which will produce the ℓ-invariants of the central ideal class group of an Abelian extension over $\boldsymbol{Q}$ for each prime ℓ dividing its order.

In the present paper we determine the central ideal class group of a cyclotomic field over $\boldsymbol{Q}$ in terms of generators and relations by refining upon the methods used in [3, 4] (§ 3, Theorem 5). The proof is based on Theorem 32 of our preceding paper [10], which is a generalization of Fröhlich [2, Theorem 3] to the case of a cyclotomic field over $\boldsymbol{Q}$.

Notation

Throughout this paper the following notation will be used.

$\boldsymbol{Q}$ the field of rational numbers as in Introduction.

$\boldsymbol{Z}$ the ring of rational integers on which a finite group acts trivially.

Received August 4, 1978.

1) Cf. Fröhlich [3, p. 212] and [4, pp. 73-77]. When $[K: \boldsymbol{Q}] = \ell^{\alpha}$, this implies that $K_0^* = K$ or $K^* = K$ according as $\ell = 2$, K real or otherwise, where K_0^* is the maximal real, unramified, Abelian 2-extension of K which is still Abelian over $\boldsymbol{Q}$, and K^* is the maximal, unramified, Abelian ℓ-extension of K which is still Abelian over $\boldsymbol{Q}$.