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Nagoya Math. J.
Vol. 75 (1979), 71-85

REMARKS TO THE UNIQUENESS PROBLEM OF MEROMORPHIC MAPS INTO $P^N(C)$, III

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§1. Introduction

In the previous papers [3], [4], [5] the author gave some results on the uniqueness of meromorphic maps of C^n into the N-dimensional complex projective space $P^N(C)$ which have the pre-assigned inverse images for some hyperplanes in $P^N(C)$. Relating to these results, we attempt in this paper to generalize the following Cartan-Nevanlinna's theorem to the case of meromorphic maps into $P^N(C)$.

THEOREM ([2], [6]). *If three non-constant meromorphic functions φ, ψ, χ on C have the same inverse images with multiplicities counted for three distinct values, then $\varphi = \psi$, $\psi = \chi$ or $\chi = \varphi$.*

Let us consider $N+2$ hyperplanes $H_1, \cdots, H_{N+2}$ in $P^N(C)$ located in general position and $N+2$ divisors $\nu_1, \cdots, \nu_{N+2}$ whose supports have no common irreducible components. Our purpose is to study the set $\mathscr{F}$ of all meromorphic maps f of C^n into $P^N(C)$ such that the pull-backs $\nu(f, H_j)$ of the divisors (H_j) on $P^N(C)$ by f are equal to ν_j $(1 \leqq j \leqq N+2)$. The main result is stated as follows:

The set $\mathscr{F}$ cannot contain more than $N+1$ algebraically independent maps (c.f., Definition 2.1).

It is possible for $\mathscr{F}$ to contain $N+1$ distinct maps. For example, taking N^2+N algebraically independent nowhere zero entire functions η_j^i $(1 \leqq i \leqq N, 1 \leqq j \leqq N+1)$ and putting

$$k_j := (-1)^{j-1} \det\left(\eta_\ell^k - 1; \begin{matrix} k = 1, \cdots, N \\ \ell = 1, \cdots, j-1, j+1, \cdots, N+1 \end{matrix}\right)$$

for each $j = 1, 2, \cdots, N+1$, we define $N+1$ meromorphic maps $f^i: C^n$

Received March 15, 1978.