M. Matsuda
Nagoya Math. J.
Vol. 74 (1979), 87-94

THE GROUP OF AUTOMORPHISMS OF A DIFFERENTIAL ALGEBRAIC FUNCTION FIELD

MICHIHIKO MATSUDA

Abstract

Consider a one-dimensional differential algebraic function field K over an algebraically closed ordinary differential field k of characteristic 0. We shall prove the following theorem:

Suppose that the group of all automorphisms of K over k is infinite. Then, K is either a differential elliptic function field over k or $K = k(v)$ with $v' = \xi$ or $v' = \eta v$, where $\xi, \eta \in k$.

It will not be assumed that the field of constants of K is the same as that of k. If we set this additional assumption, then our result is contained in a theorem due to Kolchin [4, p. 809].

§ 0. Introduction

Let k be an algebraically closed ordinary differential field of characteristic 0, and K be a one-dimensional algebraic function field over k. We shall assume that K is a differential extension of k. Then, K is called a *differential algebraic function field* over k if there exists an element y of K such that $K = k(y, y')$. Let F be an algebraically irreducible element of the differential polynomial algebra $k\{y\}$ of the first order. Then, there exists a differential algebraic function field K over k such that $K = k(y, y')$ and $F(y, y') = 0$. Throughout this note K will denote a differential algebraic function field over k.

We call K a *differential elliptic function field* over k if there exists an element z of K such that $K = k(z, z')$ and

$$(z')^2 = \lambda z(z^2 - 1)(z - \delta);\ \lambda, \delta \in k;\ \lambda \neq 0;\ \delta^2 \neq 0, 1;$$

here δ is a constant.

Received March 15, 1978.
AMS 1970 subject classifications. Primary 12H05, 14H05. Secondary 33A25, 34A05, 34A20.