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GROUPS WITH A (B, N)-PAIR AND LOCALLY TRANSITIVE GRAPHS

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1. Introduction.

Let Γ be an undirected graph and G a subgroup of aut(Γ). We denote by $\partial(x, y)$ the distance between two vertices x and y, by $E(\Gamma)$ the edge set of Γ, by $V(\Gamma)$ the vertex set of Γ, by $\Gamma(x)$ the set of neighbors of the vertex x and by $G(x)^{\Gamma(x)}$ the permutation group induced by the stabilizer $G(x)$ on $\Gamma(x)$. For each $i \in N$, let $G_i(x) = \{a \mid a \in G(y)$ for every y with $\partial(x, y) \leqslant i\}$. An s-path is an ordered sequence $(x_0, \cdots, x_s)$ of $s+1$ vertices x_i with $x_i \in \Gamma(x_{i-1})$ for $1 \leqslant i \leqslant s$ and $x_i \neq x_{i-2}$ for $2 \leqslant i \leqslant s$. For each vertex x, let $W_s(x)$ be the set of s-paths $(x_0, \cdots, x_s)$ with $x = x_0$. We say that the graph Γ is locally (G, s)-transitive if for every vertex x, $G(x)$ acts transitively on $W_s(x)$ but not on $W_{s+1}(x)$ (compare [1], [11]). If, in addition, G acts transitively on $V(\Gamma)$, then Γ is called (G, s)-transitive; otherwise Γ is bipartite with vertex blocks V_0 and V_1 and G acts transitively on both V_0 and V_1, assuming that Γ is connected and $s \geqslant 1$.

Now let G be a finite group with a (B, N)-pair whose Weyl group is a dihedral group D_{2n} of order $2n$ $(n \geqslant 2)$ and Γ be the incidence graph of the associated coset geometry as defined in [3, p. 129] (or [2, (15. 5. 1)]). The graph Γ has the following properties:

(A) $V(\Gamma) = V_0 \cup V_1$ with $V_0 \cap V_1 = \varnothing$ and $\Gamma(x) \subseteq V_{1-i}$ for every vertex $x \in V_i$ $(i = 0$ and $1)$. For $i = 0$ and 1 there exists a $d_i \in N$ such that $|\Gamma(x)| = d_i + 1$ for every vertex $x \in V_i$. The diameter of Γ is n and the girth $2n$.

(B) Γ is locally $(G, n+1)$-transitive.

A generalized n-gon of order (d_0, d_1) is, by definition, an incidence structure whose incidence graph has the properties listed in (A).

W. Feit and G. Higman have shown in [3] that finite generalized n-gons of order (d_0, d_1) with $d_0 d_1 > 1$ exist only for $n = 2, 3, 4, 6, 8$ and 12, that

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