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ON THE EXISTENCE OF CONDENSER POTENTIALS

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0. Introduction

The existence of condenser potentials was established in the framework of Dirichlet spaces by Beurling and Deny, cf. Deny [5] or Landkof [10], simply by choosing the potential of minimal energy within a certain convex set. This same idea works for non-symmetric Dirichlet spaces, cf. Bliedtner [3].

Let $(\mu_t)_{t>0}$ be a transient convolution semigroup on a locally compact abelian group G and let $\kappa = \int_0^\infty \mu_t dt$ be the potential kernel. The associated negative definite function on $\hat{G}$ is denoted ψ. For an explicit formulation of these concepts see [2]. We may now state that κ satisfies the condenser principle in the special case where κ is associated with an ordinary Dirichlet space, corresponding to ψ being realvalued, cf. [7], or more generally in the case where κ is associated with a non-symmetric Dirichlet space, corresponding to ψ satisfying the inequality $|\operatorname{Im} \psi| \leq A \operatorname{Re} \psi$, cf. [1].

The purpose of this note is to show that every potential kernel κ satisfies the condenser principle.

The condenser potential is constructed as sum of an alternating infinite series. As an application it is proved that the condenser measures are concentrated on the boundaries if and only if $(\mu_t)_{t>0}$ consists of probability measures and is of local type. Results of this type was obtained by Itô [8] for Dirichlet spaces.

A similar approach to condenser potentials but in the context of function kernels has been given by Kishi [9].

In section 3 we finally give an extension of the condenser principle to arbitrary Hunt convolution kernels.

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