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ON SUBCLASSES OF INFINITELY DIVISIBLE DISTRIBUTIONS ON *R* RELATED TO HITTING TIME DISTRIBUTIONS OF 1-DIMENSIONAL GENERALIZED DIFFUSION PROCESSES

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1. Introduction

A distribution μ on $\mathbf{R}_+ = [0, \infty)$ is said to be a CME^f_+ distribution if there are an increasing (in the strict sense) sequence of positive real numbers $\{a_k\}_{k=1}^{\ell}$ and $0 = b_0 < b_1 < \cdots < b_m < b_{m+1} = \infty$ $(0 \leqq m < \ell < \infty)$ such that, for each $j = 0, \ldots, m$, there is at least one a_k satisfying $b_j < a_k < b_{j+1}$ and the Laplace transform $\mathscr{L}\mu(s) = \int_{\mathbf{R}_+} e^{-sx}\mu(dx)$ of μ is represented as

$$
\begin{aligned}
\mathscr{L}\mu(s) &= \textstyle\prod_{i=1}^{\ell} a_i(s + a_i)^{-1} & \text{if } m = 0, \\
&= \textstyle\prod_{i=1}^{\ell} a_i(s + a_i)^{-1} / \prod_{j=1}^{\ell} b_j(s + b_j)^{-1} & \text{if } m \geqq 1.
\end{aligned}
$$

The author [8] shows that the upward first passage time distributions of birth and death processes belong to the class CME^f_+. He [9] also shows that the class of distributions of hitting times of single points of generalized diffusion processes is a proper subclass of the closure CME_+, in the weak convergence sense, of CME^f_+. Let CME^f_- be the class of distributions on $\mathbf{R}_- = (-\infty, 0]$ whose mirror images belong to CME^f_+. That is, $\mu \in CME^f_-$ if and only if $\bar{\mu}(du) = \mu(-du)$ belongs to CME^f_+. Let CME^f be the class of $\mu = \mu_1 * \mu_2$ with $\mu_1 \in CME^f_+$ and $\mu_2 \in CME^f_-$. Sato [4] shows that the distributions of sojourn times of birth and death processes with weight not necessarily positive belong to CME^f.

We denote the class of infinitely divisible distributions on $\mathbf{R}$ (or $\mathbf{R}_\pm$) by $\mathscr{I}(\mathbf{R})$ (or $\mathscr{I}(\mathbf{R}_\pm)$). The classes CME^f_+ and CME_+ are contained in $\mathscr{I}(\mathbf{R}_+)$. The class CME^f is contained in $\mathscr{I}(\mathbf{R})$. Some interesting classes of infinitely divisible distributions on $\mathbf{R}_+$ (for example, BO, CE_+, ME_+, $CME_+, \ldots$) are introduced in [1] and [8] and representations of their Laplace transforms, compactness

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