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HOLOMORPHIC FAMILIES OF GEODESIC DISCS IN INFINITE DIMENSIONAL TEICHMÜLLER SPACES

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1. Introduction

The theory of quasiconformal mappings plays an important role in Teichmüller theory. The Teichmüller spaces of Riemann surfaces are defined as quotient spaces of the spaces of Beltrami differentials, and the Teichmüller distances are defined to measure quasiconformal deformations between the Riemann surfaces representing points in the Teichmüller spaces. The Teichmüller spaces are complex Banach manifolds equipped with natural complex structures such that the canonical projections are holomorphic. It is known (see Gardinar [4]) that the Teichmüller distance, defined independently of the complex structures, coincides with the Kobayashi distance.

In spite of the naturality of the definition of a Teichmüller space as a quotient of Beltrami differentials, for given two Beltrami differentials it is hard to determine whether they are equivalent or not. For this reason, it is not trivial to describe geodesic lines with respect to the Teichmüller-Kobayashi metric.

In finite dimensional cases, each pair of points determines a unique geodesic line through them, and the geodesic line is obtained by the unique extremal differential called a Teichmüller differential. Namely, each geodesic line through the base point is represented as $\{t\mu : -1 < t < 1\}$ with a Teichmüller differential μ. On the other hand, little was known about infinite dimensional cases for a long time. In this case, it is known that there exists a point p which contains two different extremal Beltrami differentials μ_1 and μ_2, and both of the sets $\{t\mu_1/\|\mu_1\|_\infty : -1 < t < 1\}$ and $\{t\mu_2/\|\mu_2\|_\infty : -1 < t < 1\}$ are geodesic lines. However, one cannot conclude immediately that these lines are distinct, since equivalence of μ_1 and μ_2 does not imply equivalence or non-equivalence of $t\mu_1$ and $t\mu_2$ for $t \neq 1$.

Recently, L. Zhong [20] showed non-uniqueness of geodesic lines through

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