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LIMIT THEOREMS FOR STOCHASTIC DIFFERENCE-DIFFERENTIAL EQUATIONS

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1. Introduction

There are extensive works on the limit theorems for sequences of stochastic ordinary differential equations written in the form:

$$\frac{d\varphi_t}{dt} = f_t^n(\varphi_t) + \bar{g}_t^n(\varphi_t), \tag{1.1}$$

where $f_t^n = f_t^n(x)$, $t \geq 0$ is a stochastic process and $\bar{g}_t^n = \bar{g}_t^n(x)$, $t \geq 0$ is a deterministic function, both of which take values in the space of vector fields. The case where $\{f_t^n\}_n$ satisfies certain mixing conditions has been studied by Khas'minskii [7], Kesten-Papanicolaou [6] and others. The limit process is characterized as a diffusion process governed by a stochastic differential equation based on a Brownian motion. Further, the approximation theorem of stochastic differential equation studied by Wong-Zakai [18], Ikeda-Watanabe [4] etc. is also formulated in this way. A unified method of treating these problems was proposed by Kunita [9].

On the other hand, a lot of attention has also been shown to the discrete time approximation of stochastic differential equations. Approximating sequence of equations is written as

$$\varphi_{k+1} = \varphi_k + f_k^n(\varphi_k)\xi_k^n + \bar{g}_k^n(\varphi_k),\ k = 1,2,\dots \tag{1.2}$$

where $\{\xi_k^n\}$ is an array of random variables with certain mixing conditions and $\{f_k^n, \bar{g}_k^n\}_n$ is an array of continuous maps of the state space into itself. See Kushner [13], H. Watanabe [17] and Fujiwara [2]. The limit process is either a diffusion process mentioned above or a diffusion process with jumps governed by a stochastic differential equation based on a Lévy process.

In this paper we will present a unified method which is applicable both to stochastic ordinary differential equation (1.1) and to stochastic difference