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ON THE L^2 COHOMOLOGY OF COMPLEX SPACES II

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Introduction

This is a continuation of the author's previous work [O-6], in which we have settled a conjecture of Cheeger-Goresky-MacPherson [C-G-M] by proving that the L^2 cohomology group of a compact (reduced) complex space is canonically isomorphic to its (middle) intersection cohomology group. Our aim here is, in addition to that result, to extend further the classical L^2 harmonic theory to complex spaces with arbitrary singularities by establishing the following.

THEOREM 1. *Let X be a compact Kähler space and $H^r_{(2)}(X)$ its r-th L^2 cohomology group. Then every element in $H^r_{(2)}(X)$ is uniquely representable as a sum $\sum_{p+q=r} u^{p,q}$ where $u^{p,q}$ are L^2 harmonic forms of type (p, q). In particular*

$$H^r_{(2)}(X) = \bigoplus_{p+q=r} H^{p,q}_{(2),d}(X).$$

Here $H^{p,q}_{(2),d}(X)$ denotes the subspace of $H^r_{(2)}(X)$ consisting of the elements which are representable by (p, q)-forms. Moreover the complex conjugate of $H^{p,q}_{(2),d}(X)$ is equal to $H^{q,p}_{(2),d}(X)$.

Combined with our previous result, Theorem 1 implies that the intersection cohomology group of a compact Kähler space admits a canonical Hodge structure. Thus we are left with a question whether or not our (L^2-) Hodge structure coincides with another one introduced by M. Saito [S]. It follows from the works of Zucker [Z] and the author [O-5] that they coincide if X admits only isolated singularities.

As for the proof of Theorem 1, a crucial step is in establishing the existence of a family of complete Kähler metrics on $X' := X - \operatorname{Sing} X$ converging to the

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