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INFINITE DIMENSIONAL CYCLES ASSOCIATED TO OPERATORS

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§ 0. Introduction

A family of operators defined on infinite dimensional spaces gives rise to interesting cycles (or subvarieties) of infinite dimension which represent a topological or non-topological feature of operator families. In this paper we will give a general theory of these cycles, and give some estimates among them. We will apply this theory, in the final section, to cycles derived from Dirac operators.

We take, as a parameter space of operators, a paracompact space X of infinite dimension. We let $\mathscr{F} = \{\mathscr{F}_x\}_{x \in X}$ be a family of Fredholm operators parametrized by X (this family will be called *Fredholm morphism* in the sense of Elworthy-Tromba [5]). Associated to $\mathscr{F}$, there are two kinds of cycles (or subvarieties of X) that will be our main interest in this paper. One is called *solution-cycles*, which will be defined as a representation of a global structure of spaces consisting of solutions of $\mathscr{F}$. The other is called *index-cycles*, which are determined by the index (as a family) of $\mathscr{F}$. In general, the latter cycles can be calculated using the Atiyah-Singer Index Theorem for families of operators. Our aim is to estimate solution-cycles which are not invariant in general by index-cycles which are topologically invariant. In particular, we can prove a non-triviality of solution-cycles for Dirac operators using this estimate. We will denote solution-cycles by $\kappa^{r,s}_{p,q}$, and denote index-cycles by $\phi^{r,s}_{p,q}$. These cycles will turn out to be tied up with a symplectic geometry and a theory of loop groups, which will be mentioned in forthcoming publications (cf. [15]). In the present paper, we will show the existence of cycles, for a family of operators $\mathscr{F}$, satisfying:

$$|\kappa|^{r,s}_{p,q}(\mathrm{Ker}(\mathscr{F})) \supset |\phi|^{r,s}_{p,q}(\mathrm{Ind}(\mathscr{F})),$$

here $|*|$ denotes the carrier of a cycle $*$ (or a subvariety) on X (see §2). We will

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