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THE LOW DENSITY LIMIT IN FINITE TEMPERATURE CASE

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§ 1. Introduction

The low density limit in the Boson Fock case has been investigated in [1] where also the physical meaning and their motivations have been explained (cf. also [0]). From these papers one knows how the number processes can be obtained from a quantum Hamiltonian model via a certain limit procedure.

But the physically more meaningful case is the one in which the initial state of the reservoir is a finite temperature rather than Fock state. The present paper is devoted to do this case.

In the present paper, our physical model and essential assumptions are same as ones of [1] (except the Fock state). We consider a "System + Reservoir" model described by a system Hilbert space H_0 and a (one particle) reservoir Hilbert space H_1; the system Hamiltonian H_S; the (one particle) reservoir Hamiltonian Δ, and the associated one particle reservoir evolution

$$S_t^0 := \exp(-it\Delta) \tag{1.0}$$

which is a one parameter unitary group on $B(H_1)$.

Let be given, on the CCR C^*-algebra over H_1 (cf. [11]), a gauge invariant quasi-free state $\varphi^{(z)}$ with fugacity z^2, i.e. for each $f \in H_1$,

$$\varphi^{(z)}(W(f)) = \exp\left(-\frac{1}{2}\langle f, (1 + z^2 e^{-\frac{1}{2}H\beta})(1 - z^2 e^{-\frac{1}{2}H\beta})^{-1} f\rangle\right) \tag{1.1}$$

where H is a self-adjoint operator which, will be supposed in this paper, commutes with Δ and $W(f)$ is the Weyl operator with test function f. Moreover up to GNS-construction one can write the left hand side of (1.1) as

$$\langle \Phi_z, W(f)\, \Phi_z \rangle$$

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