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MODIFIED DEFECT RELATIONS FOR THE GAUSS MAP OF MINIMAL SURFACES, III

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Dedicated to Professor N. Tanaka for his 60th birthday

§ 1. Introduction

In [5], the author proved that the Gauss map of a nonflat complete minimal surface immersed in $\mathbf{R}^3$ can omit at most four points of the sphere, and in [7] he revealed some relations between this result and the defect relation in Nevanlinna theory on value distribution of meromorphic functions. Afterwards, Mo and Osserman obtained an improvement of these results in their paper [11], which asserts that if the Gauss map of a nonflat complete minimal surface M immersed in $\mathbf{R}^3$ takes on five distinct values only a finite number of times, then M has finite total curvature. The author also gave modified defect relations for holomorphic maps of a Riemann surface with a complete conformal metric into the n-dimensional complex projective space $\mathbf{P}^n(\mathbf{C})$ and, as its application, he showed that, if the (generalized) Gauss map G of a complete minimal surface M immersed in $\mathbf{R}^m$ is nondegenerate, namely, the image $G(M)$ is not contained in any hyperplane in $\mathrm{P}^{m-1}(C)$, then it can omit at most $m(m+1)/2$ hyperplanes in general position ([8]). Here, the number $m(m+1)/2$ is best-possible for arbitrary odd numbers and some small even numbers m (see [6]). Recently, Ru showed that the "nondegenerate" assumption of the above result can be dropped ([13]). In this paper, we shall introduce a new definition of modified defect and prove a refined Modified defect relation. As its application, we shall give some improvements of the above-mentioned results in [5], [7], [8], [11] and [13].

We roughly explain here the modified defect relations given in this paper. More precise statements are given in § 5. Let M be an open Riemann surface with a conformal metric ds^2 and consider a nondegenerate

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