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A DECOMPOSITION THEOREM OF 2-TYPE IMMERSIONS

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§ 1. Introduction

One branch of the research of submanifolds was introduced by Chen in terms of *type* in [2]. Type of a submanifold makes clear how the eigenspace decomposition of the Laplacian (of the ambient space) preserve after restricted to the submanifold.

We will review the definition of type of a submanifold M in the unit sphere $S^m(1)$ in the Euclidean space E^{m+1}. Let x be the canonical coordinate in E^{m+1}. We call M k-type if x is decomposed into k maps $x_1, \cdots, x_k$ such that

$$x = x_1 + \cdots + x_k ,$$
$$\Delta x_i = \lambda_i x_i \qquad \text{for } i = 1, \cdots, k$$

as a vector valued function, where Δ is the Laplacian of M. As coordinate functions generate the 1st eigenspace of $S^m(1)$, k-type means that the 1st eigenspace of $S^m(1)$ restricted to M is decomposed into k eigenspaces of M. We can generalize the definition to the k-type via l-th eigenspace of other ambient spaces in the same way. But here as we are concerned only with surfaces of 2-type in $S^m(1)$, we will not refer to it anymore. For the precise definitions, see § 5. See [1], [5] etc. for other relevant results for the general case.

The immersion ι: $M \rightarrow S^m$ is called *mass-symmetric* if the center of mass of $\iota(M)$ coincides with the center of S^m.

In terms of the type of immersions, a well known theorem of Takahashi [4] states that an n-dimensional compact submanifold M of E^{m+1} is 1-*type if and only if M is a minimal submanifold of a hypersphere S^m* of E^{m+1}, and any compact minimal submanifold of S^m is known to be mass-symmetric. Our results can be stated as follows.

THEOREM 1. *Any mass-symmetric and proper* 2-*type immersion of a*

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