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A RIESZ DECOMPOSITION THEOREM

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Introduction

The topic of this note is the Riesz decomposition of excessive functions for a "nice" strong Markov process X. I.e. an excessive function is decomposed into a sum of a potential of a measure and a "harmonic" function. Originally such decompositions were studied by G.A. Hunt [8]. In [1] a Riesz decomposition is given assuming that the state space E is locally compact with a countable base and X is a transient standard process in strong duality with another standard process $\hat{X}$ having a strong Feller resolvent. Recently R.K. Getoor and J. Glover extended the theory to the case of transient Borel right processes in weak duality [6].

In a different direction K.L. Chung and M. Rao [2] discussed the Riesz representation and other related topics without assuming duality. Their conditions are analytic ones imposed on the potential density $u(x, y)$. To be precise, they assume that $u(x, y)$ is the potential density of a transient Hunt process and satisfies:

$u(x, y)$ is extended continuous in y for any fixed x, $u(x, y) > 0$ for any (x, y) and $u(x, y) = \infty$ if and only if $x = y$.

It is proved in [2] that the Riesz decomposition holds for any excessive function. In [9] Ming Liao extends the results of Chung and Rao under slightly weaker assumptions.

The frame for this note is a transient Borel right process X on a Lusin topological space E with potential density $u(x, y)$ with respect to a given excessive reference measure m. No duality is assumed. In Section 1—using pure potential theoretic standard H-cone technique—we construct the potential part $U\mu_s$ of the Riesz decomposition of a given excessive function s. The assumption on $u(x, y)$ needed for this construction is properness and a point separating property of the dual operator $\hat{U}$ defined by

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