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A LIPMAN'S TYPE CONSTRUCTION, GLUEINGS AND COMPLETE INTEGRAL CLOSURE

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§ 0. Introduction

Given a semilocal 1-dimensional Cohen-Macauly ring A, J. Lipman in [10] gives an algorithm to obtain the integral closure $\bar{A}$ of A, in terms of prime ideals of A. More precisely, he shows that there exists a sequence of rings $A = A_0 \subset A_1 \subset \cdots \subset A_i \subset \cdots$, where, for each i, $i \geq 0$, A_{i+1} is the ring obtained from A_i by "blowing-up" the Jacobson radical $\mathscr{R}_i$ of A_i, i.e. $A_{i+1} = \cup_n (\mathscr{R}_i^n : \mathscr{R}_i^n)$. It turns out that $\cup\{A_i; i \geq 0\} = \bar{A}$ (cf. [10, proof of Theorem 4.6]) and, if $\bar{A}$ is a finitely generated A-module, the sequence $\{A_i;\ i \geq 0\}$ is stationary for some m and $A_m = \bar{A}$, so that

$$(+) \qquad A = A_0 \subsetneqq A_1 \subsetneqq \cdots \subsetneqq A_m = \bar{A}\,.$$

In [15] G. Tamone studies when in the Lipman's sequence (+) A_i is a "glueing of primary ideals of A_{i+1} over a prime ideal of A" (see [14] for definition). She shows in particular that A_i is not always a glueing of primary ideals of A_{i+1}.

In this paper we give an algorithmic construction, for a Noetherian domain A of any dimension, such that $\bar{A}$ is a finitely generated A-module, defining a new sequence $\{A_i;\ i \geq 0\}$ of overrings of A; A_{i+1} is obtained from A_i, taking the dual of a distinguished radical ideal of A_i. We show that such a sequence is stationary for some m, $A_m = \bar{A}$ (cf. Theorem 1.8), and A_i is always a glueing of primary ideals of A_{i+1} (cf. Proposition 2.7 and Remark 2.2, a)).

A similar sequence was considered in [17] by K. Yoshida in the case of a Noetherian ring satisfying the S_1-condition. As a matter of fact, the intermediate rings of the Yoshida sequence are defined in a rather different way, but the prime ideals occuring in their definition are linked to those that we use in our sequence (cf. for more details Remark 1.7).

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