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GENERAL SOLUTIONS DEPENDING ALGEBRAICALLY ON ARBITRARY CONSTANTS

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§ 1. Introduction

In his famous lectures [7] Painlevé investigates general solutions of algebraic differential equations which depend algebraically on some of arbitrary constants. Although his discussions are beyond our understanding, the rigorous and accurate interpretation to make his intuition true would be possible. Successful accomplishments have been done by some authors, for example, Kimura [1], Umemura [8, 9]. From differential algebraic viewpoint in [5] the author introduces the notion of rational dependence on arbitrary constants of general solutions of algebraic differential equations, and in [6] clarifies the relation between it and the notion of strong normality. Here we aim at generalizing to higher order case the result in [4] that in the first order case solutions of equations depend algebraically on those of equations free from moving singularities which are determined uniquely as the closest ones to the given. Part of our result can be seen in [7].

Let K be an algebraically closed ordinary differential field of characteristic zero. In what follows we tacitly assume every differential field extension of K is a finitely generated one and is contained in a fixed universal differential field extension of K. In order to reckon the degree of algebraic dependence of a differentially algebraic element, we begin with explaining an ordinary ordering among multi-indices. By a multi-index we mean a sequence $J = (j_n)$, where n runs through all nonnegative integers and the j's are nonnegative integers, being zero except for a finite number of the j's. Let $I = (i_n)$ and $J = (j_n)$ be two multi-indices. We say I is lower than J or J is higher than I if there is an integer m with $i_m < j_m$ and $i_n = j_n$ for all $n > m$. Let E be a differential field extension of K and $E\{Y\}$ denote the algebra of differential polynomials

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