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ULTRAPOTENTIALS AND POSITIVE EIGENFUNCTIONS FOR AN ABSOLUTELY CONTINUOUS RESOLVENT OF KERNELS

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Introduction

Let $(X, \mathscr{B})$ be a measurable space and $\mathscr{V}$ be a submarkovian resolvent of kernels (with the initial kernel V proper) on X which is absolutely continuous and has a dual resolvent (with the same properties) with respect to a σ-finite measure.

A positive numerical function s on X is called *V-ultrapotential* if it is $\mathscr{V}$-excessive (in particular $\mathscr{V}$-a.e. finite) and if the following condition is fulfilled: for every integer $n \geq 1$, there exists a positive $\mathscr{B}$-measurable function f_n on X such that $s = V^n(f_n)$, where V^n is the n-th iteration of the kernel V.

The main purpose of this paper (see Theorem 3.5 and Corollary 3.6) is to prove that, under a "regularity" condition (which will be discussed in the last part of Section 2) on the resolvent $\mathscr{V}$, for each V-ultrapotential s there exist a finite positive Borel measure σ on the open interval $]0, \infty[$ and a family $(s_\lambda)_{0<\lambda<\infty}$, s_λ being a positive λ-eigenfunction of V (i.e. $V(s_\lambda) = \lambda \cdot s_\lambda$ and s_λ is $\mathscr{V}$-a.e. finite), for any $\lambda > 0$, such that for each $x \in X$ the numerical function $\lambda \mapsto s_\lambda(x)$, defined on $]0, \infty[$, is σ-measurable and

$$s(x) = \int s_\lambda(x) d\sigma(\lambda) .$$

In fact, this type of representation is given for a slightly more general class of excessive functions, as the V-ultrapotentials.

An uniqueness of the representation and a converse statement are also proved.

These results are analogous, in this context, with those obtained by M. Itô and N. Suzuki in [7] (see also [6]) for the set up of diffusion semi-

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