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UNIT THEOREMS ON ALGEBRAIC TORI

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Let k be a p-adic field (a finite extension of $\boldsymbol{Q}_p$) or an algebraic number field (a finite extension of $\boldsymbol{Q}$). Let T be an algebraic torus defined over k. We denote by $\hat{T}$ the character module of T (i.e. $\hat{T} = \mathrm{Hom}(T, G_m)$), where G_m is the multiplicative group.

As is well-known (cf. [7]), T is split by a finite galois extension K/k. We denote by G the galois group of K/k. Then $\hat{T}$ becomes naturally a G-module. Since the map $T \to \hat{T}$ yields a canonical isomorphism between the category of tori defined over k and split by K and the dual category of finitely generated $\boldsymbol{Z}$-free G-modules, it is natural to use $\mathrm{Hom}_G(\hat{T}, M_K)$ as a definition of an object relative to T over k when M_K is a G-module of arithmetical interest related to K.

In this paper, we will determine the structure of $\mathrm{Hom}_G(\hat{T}, O_K^\times)$ where $O_K^\times$ is the group of units of K and will discuss the meaning of this group.

§1. Local unit theorem

Let k be a p-adic field. First we recall the structure of $O_k^\times$. Let π be a prime element of k and let U_1 be the group of one units of k i.e. $U_1 = 1 + \pi O_k$. $\boldsymbol{Z}_p$ acts on U_1 as follows:

Let $a = a_0 + a_1 p + \cdots + a_n p^n + \cdots \in \boldsymbol{Z}_p$ and $u \in U_1$. Set $a_n = \sum_{i=0}^n a_i p^i$. Then $\{u^{a_n}\}$ is a Cauchy sequence in U_1. Since U_1 is compact, the limit exists and denoted by u^a.

So we can view U_1 as $\boldsymbol{Z}_p$-module. We have the following proposition (cf. [5]).

(1.1) PROPOSITION. $U_1 \approx W(U_1) \times \boldsymbol{Z}_p^{[k \cdot Q_p]}$, *where* $W(U_1)$ *is the group of roots of unity in* U_1. □

Now $O_k/(\pi)$ has $q = p^s$ elements. Let η be a primitive $(q-1)$th root of unity in O_k. Then

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