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MODULAR FORMS AND THE AUTOMORPHISM GROUP OF LEECH LATTICE

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Dedicated to Professor Michio Kuga on his 60th birthday

This is a continuation of my previous papers [2], [3], [4] concerning to the monstrous moonshine.

The automorphism group $\cdot O$ of the Leech lattice L plays an important role in the study of moonshine. Especially it is important to study theta functions associated with quadratic sublattices of L consisting of fixed vectors of elements of $\cdot O$. In this paper, we discuss the properties that these functions are expected to satisfy in the relation to the monstrous moonshine.

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§1.

1.1. Throughout this paper, we use the same notation as in [4] and [5]. We recall them first.

$\cdot O$ has a natural 24-dimensional representation over $\boldsymbol{Q}$ induced by the action on the Leech lattice. So each element π of $\cdot O$ is described by Frame shape with respect to this representation:

$$\pi = \prod_{1 \leqq t} t^{r_t}, \qquad r_t \in Z.$$

Then $\deg \pi = \sum t \cdot r_t = 24$. Let $\mathrm{wt}\, \pi = \frac{1}{2} \sum r_t$. We classify every elements of $\cdot O$ into the following 3 types:

(1.1) π is called type C if $r_t \geqq 0$ for all $t \geqq 1$.

(1.2) π is called type E if $\mathrm{wt}\, \pi$ is positive but there exists some t such that $r_t < 0$.

(1.3) π is called type F if $\mathrm{wt}\, \pi = 0$.

For each π, we can suitably choose a positive integer N which is a

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