

A MODIFICATION OF GROTHENDIECK'S SPECTRAL SEQUENCE

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Introduction

Let C , C' and C'' be abelian categories where C and C' have enough injectives and let $F: C \rightarrow C'$, $G: C' \rightarrow C''$ be additive covariant functors. Then for an object X of C , let $C(X)$ be the complex associated with an injective resolution of X . Grothendieck gets a first quadrant spectral sequence by taking an injective resolution of the complex $F(C(X))$ and applying G to the associated double complex. Under certain hypotheses one gets a spectral sequence

$$E_2^{pq} = R^p G(R^q F(X)) \Rightarrow R^n(GF)(X) .$$

If we modify this procedure by replacing $C(X)$ with a projective resolution of X and then proceed as above, we get a second quadrant spectral sequence. Using these spectral sequences, a variety of known results can be proved and sharpened.

In the first applications $C = C'$ and $F = \text{id}_C$, so initially to simplify notation make this assumption (Grothendieck's spectral sequence becomes trivial in this case). Some applications will require slight changes in these hypotheses, but it will then be easy to see how to modify the proofs.

§1. The spectral sequence

In this paper we adopt the convention that the derived functors $R^q T = 0$ when $q < 0$ and similarly $L_p T = 0$ for $p < 0$. Also Betti and Bass numbers with strictly negative subscripts will be taken to be 0.

PROPOSITION 1.1. *Let $T: C \rightarrow C'$ be an additive functor where C and C' are abelian categories and C has enough injectives and projectives. Then for an object X of C there is a second quadrant spectral sequence*

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