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CYLINDRICAL PROBABILITIES AND THE DIFFERENTIATION OF VECTOR MEASURES

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§ 1. Introduction

There are many results in probability theory on vector spaces which rely implicitly on the approximation of a given cylindrical probability by cylindrical probabilities with moments; for example, this is the basic idea behind the proof of the Radon equivalence of the weak and strong topologies of a metrizable space (Schwartz [13] p. 162). The technique of approximation by cylindrical measures with moments can be systematically developed. In particular, it follows that if each member of a family of cylindrical probabilities with moments is decomposable, then the limits of these cylindrical probabilities are often regular and so they are σ-additive.

An application of this method is related to the work of L. Schwartz [11], [12] dealing with the notion of "p-radonifying" maps. The well-known theorem due to Sazanov (see [13]) asserts that a continuous linear operator between Hilbert spaces is Hilbert-Schmidt if and only if it maps every cylindrical probability scalarly concentrated on bounded sets into a Radon measure (i.e. it is "o-radonifying"). However, there are maps between Banach spaces which are even *nuclear* but not o-radonifying. In fact, the maps need only be a little better than nuclear maps to be o-radonifying. Furthermore, there are absolutely summing maps between Banach space which are not 1-radonifying. This phenomenon is related to the Radon-Nikodým property of the range space [12].

Since nuclear and absolutely summing maps send vector measures into indefinite integrals (i.e. they are "Nikodýmising"), the approximation argument mentioned above allows us to find classes of cylindrical probabilities for which nuclear and absolutely summing maps *are* regularizing. The method has the advantage of relying only on the notions of summability and boundedness in a locally convex space, so there is no need to

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