T. Hiramatsu and S. Akiyama
Nagoya Math. J.
Vol. 111 (1988), 157-163

ON SOME DIMENSION FORMULA FOR AUTOMORPHIC FORMS OF WEIGHT ONE III

TOYOKAZU HIRAMATSU AND SHIGEKI AKIYAMA

Dedicated to Professor Michio Kuga on his 60th birthday

Let Γ be a fuchsian group of the first kind and assume that Γ does not contain the element $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Let $S_1(\Gamma)$ be the linear space of cusp forms of weight 1 on the group Γ and denote by d_1 the dimension of the space $S_1(\Gamma)$. When the group Γ has a compact fundamental domain, we have obtained the following (Hiramatsu [3]):

$$(*) \qquad d_1 = \frac{1}{2} \operatorname*{Res}_{s=0} \zeta^*(s)\,,$$

where $\zeta^*(s)$ denotes the Selberg type zeta function defined by

$$\zeta^*(s) = \sum_{\alpha=1}^{\infty} \sum_{k=1}^{\infty} \frac{(\operatorname{sgn} \lambda_{0,\alpha})^k \log|\lambda_{0,\alpha}|}{|\lambda_{0,\alpha}^k - \lambda_{0,\alpha}^{-k}|} |\lambda_{0,\alpha}^k + \lambda_{0,\alpha}^{-k}|^{-s}\,.$$

Here, $\lambda_{0,\alpha}$ denotes the eigenvalue ($|\lambda_{0,\alpha}| > 1$) of representative P_α of the primitive hyperbolic conjugacy classes $\{P_\alpha\}$ in Γ.

In this paper we give a formula of the dimension d_1 for a general discontinuous group Γ of finite type such that $\Gamma \not\ni \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, by using the Selberg trace formula (Selberg [5], Kubota [4]).[1)]

The notation used here will generally be those of [1].

§ 1. The Selberg eigenspace $\mathcal{M}(1, -\frac{3}{2})$, Eisenstein series and continuous spectrum

1.1. Let Γ be a fuchsian group of the first kind not containing the element $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, and suppose that Γ has a non-compact fundamental

Received February 9, 1987.

1) For the dimension d_1 in the case of $\Gamma \ni \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, refer to Hiramatsu [2].