T. Kurokawa
Nagoya Math. J.
Vol. 111 (1988), 1-11

INTEGRAL REPRESENTATION OF SMOOTH FUNCTIONS IN WEIGHT CLASSES AND ITS APPLICATION

TAKAHIDE KUROKAWA

§ 1. Introduction

Let $\boldsymbol{R}^n$ be the n-dimensional Euclidean space, and for each point $x = (x_1, \cdots, x_n)$ we write $|x| = (x_1^2 + \cdots + x_n^2)^{1/2}$. For a multi-index $\alpha = (\alpha_1, \cdots, \alpha_n)$, we denote by x^α the monomial $x_1^{\alpha_1} \cdots x_n^{\alpha_n}$, which has degree $|\alpha| = \sum_{j=1}^n \alpha_j$. Similarly, if $D_j = \partial/\partial x_j$ for $1 \leqq j \leqq n$, then

$$D^\alpha = D_1^{\alpha_1} \cdots D_n^{\alpha_n}$$

denotes a differential operator of order $|\alpha|$. We also write $\alpha! = \alpha_1! \cdots \alpha_n!$. Throughout this paper, let $1 < p < \infty$ and $(1/p) + (1/p') = 1$. For a real number r, we denote by $L^{p,r}$ the class of all measurable functions f for which

$$\|f\|_{p,r} = \left(\int |f(x)(1 + |x|)^r|^p dx\right)^{1/p} < \infty .$$

The notation $\mathscr{D}$ denotes the LF-space consisting of all C^∞-functions with compact support. The symbol $\mathscr{D}'$ stands for the topological dual of $\mathscr{D}$. Let m be a positive integer. We denote by $L_m^{p,r}$ the space of all $u \in \mathscr{D}'$ such that $D^\alpha u \in L^{p,r}$ for any α with $|\alpha| = m$. We set

$$|u|_{m;p,r} = \sum_{|\alpha|=m} \|D^\alpha u\|_{p,r} .$$

If u belongs to $\mathscr{D}$, then u can be represented by its partial derivatives of m-th order as follows (Yu.G. Reshetnyak [4]):

$$u(x) = \sum_{|\alpha|=m} \frac{m}{\sigma_n \alpha!} \int \frac{(x-y)^\alpha}{|x-y|^n} D^\alpha u(y) dy ,$$

where σ_n denotes the surface area of the unit sphere. In this paper, we are concerned with integral representation of $u \in C^\infty \cap L_m^{p,r}$ and its integral

Received July 13, 1985.
Revised May 29, 1987.