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THE FEYNMAN INTEGRAL OF QUADRATIC POTENTIALS DEPENDING ON n TIME VARIABLES

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§ 1. Introduction

Let $C_1[0, T]$ denote (one-parameter) Wiener space; that is the space of continuous functions x on $[0, T]$ such that $x(0) = 0$. In a recent expository essay [21], Nelson calls attention to some functions on Wiener space which were discussed in the book of Feynman and Hibbs [13, section 3–10] and in Feynman's original paper [12, section 13]. These functions have the form

$$G(x) = \exp\left\{\int_0^T \int_0^T W(s_1, s_2; x(s_1), x(s_2))\, ds_1 ds_2\right\}. \tag{1.1}$$

Feynman obtained such functions by integrating out the oscillator coordinates in a system involving a harmonic oscillator interacting with a particle moving in a potential. Further functions like (1.1) but involving multiple integrals of more dimensions than two arise when more particles are involved; the study of such functions is the topic of this paper. In particular we consider the case where the function $W\colon [0, T]^n \times \boldsymbol{R}^n \to \boldsymbol{C}$ is quadratic in the space variables.

In [8], Chang, Johnson and Skoug consider functions on Wiener space of the form

$$F(x) = \exp\left\{-\int_0^T \int_0^T \langle A(s_1, s_2)(x(s_1), x(s_2)), (x(s_1), x(s_2))\rangle ds_1 ds_2\right\} \tag{1.2}$$

where $\{A(s_1, s_2) = (a_{ij}(s_1, s_2))\colon (s_1, s_2) \in [0, T]^2\}$ is a commutative family of 2 by 2 real, symmetric, nonnegative definite matrices with their eigenvalues $p_1(s_1, s_2)$ and $p_2(s_1, s_2)$ having square roots which are of bounded variation on the rectangle $[0, T]^2$. They showed that such functions F are in the Banach algebra S of functions on Wiener space which was introduced by

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