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Nagoya Math. J.
Vol. 110 (1988), 129-135

THE GROWTH OF THE POSITIVE SOLUTIONS OF $Lu=0$ NEAR THE BOUNDARY OF AN INNER NTA DOMAIN

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§ 1. Introduction

Let D be a bounded domain in the Euclidean space $\boldsymbol{R}^n$ ($n \geqq 2$) and L a uniformly elliptic partial differential operator of second order with α-Hölder continuous coefficients ($0 < \alpha \leqq 1$) on D.

According to N. Suzuki [3], D is said to be associated with the cone of angle $\theta < \pi/2$ if there exist positive constants h, d_0 and $K_0 \geqq 1$ such that:

(i) For any $z \in \partial D$, there exists $e_z \in \boldsymbol{R}^n$ with $|e_z| = 1$ such that $\Gamma_\theta(z, e_z) \subset D$, where $\Gamma_\theta(z, e_z)$ is the half cone obtained from $\{x \in \boldsymbol{R}^n;\ \sqrt{x_2^2 + \cdots + x_n^2} < x_1 \tan\theta,\ 0 < x_1 < h\}$ by the translation z and the rotation e_z.

(ii) Put $A_D = \{y = z + te_z \in \boldsymbol{R}^n;\ z \in \partial D,\ 0 < t < h/2\}$. Then for any $x \in D$ with $d(x) \leqq d_0$, there exist $y_x \in A_D$ and a polygonal line L_x from x to y_x such that $d(x) \leqq d(y_x)$ and the length of L_x is $\leqq K_0 d(L_x, \partial D)$.

In [4] he proved the following result:

If D is associated with a cone, there exist constants $m, m' \geqq 1$ such that for any positive solution of $Lu = 0$ in D,

$$C_u^{-1}(d(x))^m \leqq u(x) \leqq C_u(d(x))^{-m'} \tag{1}$$

with some constant $C_u \geqq 1$ depending on u, where $d(x)$ denotes the distance between x and ∂D, the boundary of D. In this paper, we shall define inner NTA (non-tangentially accessible) domains and show that for an inner NTA domain, we can choose two positive constants $m, m' \geqq 1$ satisfying (1) for all positive solutions of $Lu = 0$ in D. This is a direct extension of N. Suzuki's result. As applications of our main result, we shall establish the uniqueness theorem for L-superharmonic functions on an inner NTA domain and the Harnack inequality for inner NTA domains.

The author would like to thank Professor M. Itô for his encouragement and helpful suggestions.

Received September 5, 1986.