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ITO'S FORMULA AND LÉVY'S LAPLACIAN

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§1. Introduction

The class of normal functionals

$$\int \cdots \int_{R^n} f(x_1, \cdots, x_n) : \dot{B}_{x_1}^{p_1} \cdots \dot{B}_{x_n}^{p_n} : dx_1 \cdots dx_n \,, \qquad f \in L^1(\boldsymbol{R}^n),$$
$$(p_1, \cdots, p_n) \in (\boldsymbol{N} \cup \{0\})^n \,,$$

is, as is well known, adapted to the domain of Lévy's Laplacian and plays important roles in the works by P. Lévy and T. Hida (cf. [1], [2] and [8]), where $\dot{B}_x$ denotes one-dimensional parameter white noise and $:\dot{B}_{x_1}^{p_1} \cdots \dot{B}_{x_n}^{p_n}:$ denotes the renormalization of $\dot{B}_{x_1}^{p_1} \cdots \dot{B}_{x_n}^{p_n}$.

We are interested in a generalization of this class to that of generalized functionals of two-dimensional parameter white noise $\{W(t, x); (t, x) \in \boldsymbol{R}^2\}$, which is a generalized stochastic process with the characteristic functional

$$C(\xi) = E(\exp\{i\langle W, \xi\rangle\}) = \exp\left\{-\frac{1}{2}\|\xi\|^2\right\}, \qquad \xi \in S(\boldsymbol{R}^2).$$

As in the case [1], we are able to introduce, in Section 2, a space $(L^2)^{(-\alpha)}$ of generalized functionals and the $\mathscr{S}$-transform on $(L^2)^{(\alpha)}$ for every $\alpha > 0$. Then the calculus in terms of the white noise $W(t, x)$ will quickly be discussed.

The main purpose of this paper is to investigate how Lévy's Laplacian appears in Itô's formula for generalized Brownian functionals depending on t. To this end we first discuss a class of generalized Brownian functionals, often without any renormalization, having interest in its own right. For instance, a monomial $B_x(t)^p$ is sometimes more significant rather than the renormalized quantity $:B_x(t)^p: \equiv :\left\{\int_0^t W(r, x)dr\right\}^p:$ which is living in $(L^2)^{(-\alpha)}$. We are therefore led to construct a new space $[\![L^2]\!]^{(-\alpha)}$,

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