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A CALCULUS APPROACH TO HYPERFUNCTIONS I

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To the memory of C. Goulaouic

Introduction

In this paper, we shall give a new characterization of hyperfunctions without algebraic method and apply to give simpler proofs to problems discussed in [3], Chapter 9. In [3], the spaces of hyperfunctions $A'(K)$ with compact support in $K \Subset R^n$ $(n \geqq 1)$ is considered as the dual of the space $A(K)$ of functions which are real analytic near K. Each element u of $A'(K)$ is characterized as a density of a double layer potential in $R^n \times R$.

We shall first give a simpler characterization of the element u of $A'(K)$ in Theorem 1.2. We will consider $u \in A'(K)$ as an initial value of a unique solution of the heat equation

$$(\partial/\partial t - \Delta)U(x, t) = 0 \qquad \text{in } R^{n+1}_+ = R^n \times R_+ .$$

The regularity of $u = U(\cdot, 0)$ is described by the asymptotic behavior of U as $t \to 0$. In fact, we characterize in Theorem 1.2 the asymptotic behavior so that $u \in A'(K)$. We will see Schwartz and ultradistributions are characterized by the asymptotic behavior of U at the same time. The advantage of our approach is to unify the theory of distributions and hyperfunctions as well as to simplify proofs of important results. For example, based on Theorem 1.2, we significantly simplify the proof of Paley-Wiener-Schwartz theorem for hyperfunctions as well as Schwartz distributions in Section 2. We discuss in Section 3 hypoellipticity of a pseudodifferential equation

$$a(x, D)u = f \qquad \text{in } \Omega \subset R^n ,$$

where $u \in A'(K)$, $K \Subset \Omega$, f is a hyperfunction on Ω and $a(x, D)$ is analytic pseudodifferential operator considered in [1], [9] and [10]. Analytic and

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