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Nagoya Math. J.
Vol. 106 (1987), 79-90

THE DIRICHLET PROBLEM AT INFINITY AND COMPLEX ANALYSIS ON HADAMARD MANIFOLDS

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Introduction

In this paper we shall study hyperbolicity of Hadamard manifolds.

In Section 1 we shall define and solve the Dirichlet problem at infinity for Laplacian Δ, which gives a partial extension of the result of Anderson [1] and Sullivan [15] in Theorem 1 (cf. [4]). In Section 2 we apply the solution of the Dirichlet problem at infinity to a complex analysis on a Kähler Hadamard manifold whose metric restricted to every geodesic sphere is conformal to that of the standard sphere. It seems that the sphere at infinity of such a manifold admits a CR-structure. In fact we can define a CR-function at infinity on the sphere at infinity. We shall show in Theorem 2 that there exists a holomorphic extension from the sphere at infinity and it coincides with the solution of the Dirichlet problem at infinity, if the Dirichlet problem at infinity is solvable. So we see that such a manifold admits many bounded holomorphic functions. By the similar method we shall show in Theorem 3 that such a manifold is biholomorphic to a strictly pseudoconvex domain in C^n, if the holomorphic sectional curvature $K_h(x)$ is less than $-1/(1+r(x)^2)$, where $r(x)$ is a distance function from a pole. Theorem 3 is a partial answer to a conjecture raised by Green and Wu [8].

§ 1. Dirichlet problem at infinity

Let M be a Riemannian manifold of dimension n with metric g_{ij}. We denote by T_pM the tangent space at $p \in M$. For a C^2 function u, we define the Hessian D^2u of u at p by

$$D^2u(X, Y) = X(Yu) - (D_X Y)u$$

for $X, Y \in T_pM$, where D_x is the covariant derivative. The Laplacian Δu of u is the trace of D^2u, which is expressed by

$$\Delta u = \sum_{ij} g^{-1/2} \cdot \partial/\partial x_j \qquad (g^{1/2} g^{ij} \partial u/\partial x_i)$$