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WIENER–HOPF EQUATION AND FREDHOLM PROPERTY OF THE GOURSAT PROBLEM IN GEVREY SPACE

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Dedicated to Professor Yoshio Kato on the occasion of his 60th birthday

0. Introduction

In the study of ordinary differential equations, Malgrange ([Ma]) and Ramis ([R1], [R2]) established index theorem in (formal) Gevrey spaces, and the notion of irregularity was nicely defined for the study of irregular points. In their studies, a Newton polygon has a great advantage to describe and understand the results in visual form. From this point of view, Miyake ([M2], [M3], [MH]) studied linear partial differential operators on (formal) Gevrey spaces and proved analogous results, and showed the validity of Newton polygon in the study of partial differential equations (see also [Yn]).

The purpose of this paper is to extend results in [M3], where spectral property in Gevrey spaces of a special integro-differential operator was studied, which is induced from the Goursat problem formulated from an interior point of a side of Newton polygon defined by (0.2) below. Precisely, we characterize Fredholm property of the Goursat problem by employing the theory of (finite section) Wiener-Hopf equations, and show that such a property depends deeply on the Hilbert factorizability of Toeplitz symbol associated with the Gevrey index. We note that Fredholm property of the Goursat problem in the category of local holomorphic functions was firstly pointed out by Leray ([L]) by an typical example of operators, and a systematic study of such property is firstly done in this paper.

In order to illustrate our intention, we shall show a typical result which follows from Theorem 1 in Section 1.

Let $P \equiv P(t, x\,; D_t, D_x)$ be a linear partial differential operator of finite order with holomorphic coefficients in a neighbourhood of the origin of $\mathbf{C}_t \times \mathbf{C}_x$, and write it in the form,

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