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AFFINE HYPERSURFACES WITH PARALLEL CUBIC FORM

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§1. Introduction

As is well known, there exists a canonical transversal vector field on a non-degenerate affine hypersurface M. This vector field is called the affine normal. The second fundamental form associated to this affine normal is called the affine metric. If M is locally strongly convex, then this affine metric is a Riemannian metric. And also, using the affine normal and the Gauss formula one can introduce an affine connection ∇ on M which is called the induced affine connection. Thus there are in general two different connections on M: one is the induced connection ∇ and the other is the Levi Civita connection $\hat{\nabla}$ of the affine metric h. The difference tensor K is defined by $K(X, Y) = K_X Y = \nabla_X Y - \hat{\nabla}_X Y$. The cubic form C is defined by $C = \nabla h$ and is related to the difference tensor by

$$h(K_X Y, Z) = -\frac{1}{2} C(X, Y, Z).$$

The classical Berwald theorem states that C vanishes identically on M, implying that the two connections coincide, if and only if M is an open part of a nondegenerate quadric.

In this paper we will consider the condition $\hat{\nabla} C = 0$ for a 4-dimensional locally strongly convex affine hypersurface in $\mathbf{R}^5$. Clearly $\hat{\nabla} C = 0$ if and only if $\hat{\nabla} K = 0$. For surfaces this condition has been studied by M. Magid and K. Nomizu in [MN], where they proved the following;

THEOREM A [MN]. *Let M^2 be an affine surface in $\mathbf{R}^3$ with $\hat{\nabla} C = 0$. Then either M is an open part of a nondegenerate quadric (i.e. $C = 0$) or M is affine equivalent to an open part of the following surfaces:*

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