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COMPLEXES OF COUSIN TYPE AND MODULES OF GENERALIZED FRACTIONS

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0. Introduction

Let $\mathbf{R}$ be a commutative (Noetherian) ring, $\mathbf{M}$ an $\mathbf{R}$-module and let $\mathscr{F} = (\mathbf{F}_i)_{i\geq 0}$ be a filtration of $\mathrm{Spec}(\mathbf{R})$ which admits $\mathbf{M}$.

A complex of $\mathbf{R}$-modules is said to be of Cousin type if it satisfies the four conditions of ([GO], 3.2) which are reproduced below (Definition (1.5)). In ([RSZ], 3.4), Riley, Sharp and Zakeri proved that the complex, which is constructed from a chain of special triangular subsets defined in terms of $\mathscr{F}$ (Example (1.3)(3)), is of Cousin type for $\mathbf{M}$ with respect to $\mathscr{F}$ (Corollary (3.5)(2)). Gibson and O'carroll ([GO], 3.6) showed that the complex, which is obtained by means of a chain $\mathscr{U} = (\mathbf{U}_i)_{i\geq 1}$ of saturated triangular subsets and the filtration $\mathscr{G} = (\mathbf{G}_i)_{i\geq 0}$ induced by $\mathscr{U}$ and $\mathbf{M}$, is of Cousin type for $\mathbf{M}$ with respect to $\mathscr{G}$ (Corollary (3.5)(3)).

The purpose of this paper is to show that, when the complex is defined by a chain of triangular subsets, one can give a simpler criterion, consisting of only two conditions, for being of Cousin type (Theorem (3.1) and Corollary (3.2)). In fact, we prove that, for every complex induced by a chain of triangular subsets, the first and the second conditions of the definition of Cousin type hold (Remark (2.5)).

In ([RSZ], 3.3), Riley, Sharp and Zakeri proved that every complex of Cousin type for $\mathbf{M}$ with respect to $\mathscr{F}$ is isomorphic to the Cousin complex. Hence when we investigate the structure of a complex of Cousin type, it is useful to study the complex $\mathbf{C}(\mathscr{U}, \mathbf{M})$ of Cousin type which is constructed from special modules of generalized fractions (Corollary (3.5)) whose properties are well known.

We also get a refinement of the Exactness theorem ([SZ2], 3.3 and [O], 3.1) in our Proposition (2.13).

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