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A QUESTION OF GROSS AND THE UNIQUENESS OF ENTIRE FUNCTIONS

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1. Introduction and main results

For any set S and any entire function f let

$$E_f(S) = \bigcup_{a \in S} \{z \mid f(z) - a = 0\},$$

where each zero of $f - a$ with multiplicity m is repeated m times in $E_f(S)$ (cf. [1]). It is assumed that the reader is familiar with the notations of the Nevanlinna Theory (see, for example, [2]). It will be convenient to let E denote any set of finite linear measure on $0 < r < \infty$, not necessarily the same at each occurrence. We denote by $S(r, f)$ any quantity satisfying $S(r, f) = o(T(r, f))$ $(r \to \infty, r \notin E)$.

In 1976 Gross proved [3] that there exist three finite sets S_j $(j = 1,2,3)$, such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1,2,3$ must be identical. In the same paper Gross posed the following open question (Question 6): can one find two (or possible even one) finite set S_j $(j = 1,2)$ such that any two entire functions f and g satisfying $E_f(S_j) = E_g(S_j)$ $(j = 1,2)$ must be identical ?

The present author [4] proved the following result which is partial answer of the above question.

THEOREM A. *Let $S_1 = \{w \mid (w - a)^n - b^n = 0\}$, $S_2 = \{c\}$, where $n > 4$, a, b and c are constants such that $b \neq 0$, $c \neq a$ and $(c - a)^{2n} \neq b^{2n}$. Suppose that f and g are nonconstant entire functions satisfying $E_f(S_j) = E_g(S_j)$ for $j = 1,2$. Then $f \equiv g$.*

The set S such that for any two nonconstant entire funstions f and g the condition $E_f(S) = E_g(S)$ implies $f \equiv g$ is called a unique range set (URS in brief) of

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