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THE h-VECTOR OF A GORENSTEIN CODIMENSION THREE DOMAIN

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Let k be an infinite field and A a *standard G-algebra*. This means that there exists a positive integer n such that $A = R/I$ where R is the polynomial ring $R := k[X_1, \ldots, X_n]$ and I is an homogeneous ideal of R. Thus the additive group of A has a direct sum decomposition $A = \bigoplus A_t$ where $A_iA_j \subseteq A_{i+j}$. Hence, for every $t \geq 0$, A_t is a finite-dimensional vector space over k. The *Hilbert Function* of A is defined by

$$H_A(t) := \dim_k(A_t), \quad t \geq 0.$$

The generating function of this numerical function is the formal power series

$$P_A(z) := \sum_{t \geq 0} H_A(t) z^t.$$

As a consequence of the Hilbert-Serre theorem we can write

$$P_A(z) = h_A(z)/(1-z)^d$$

where $h_A(z) \in \mathbf{Z}[z]$ is a polynomial with integer coefficients such that $h_A(1) \neq 0$. Moreover d is the Krull dimension of the ring A.

The polynomial $h_A(z)$ is called the *h-polynomial* of A; if $h_A(z) = 1 + a_1 z + \cdots + a_s z^s$ with $a_s \neq 0$, then we say that the vector $(1, a_1, \ldots, a_s)$ is the *h-vector* of A. It is clear that the h-vector of A together with its Krull dimension determines the Hilbert Function of A and conversely.

A classical result of Macaulay gives an explicit numerical characterization of the *admissible* numerical functions, i.e. of the functions $H : \mathbf{N} \to \mathbf{N}$ which are the Hilbert Function of some standard G-algebra A. This result proved in [M] has been recently revisited by Stanley in [S]. One can easily find similar characterizations for reduced or Cohen-Macaulay G-algebras (see [GMR] and [S]).

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