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Nagoya Math. J.
Vol. 141 (1996), 125–142

# ON THE SERIES FOR $L(1, \chi)$

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## 1. Introduction

Let $k$ be a positive integer greater than 1, and let $\chi(n)$ be a real primitive character modulo $k$, The series

$$L(1, \chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n}$$

can be divided into groups of $k$ consecutive terms. Let $v$ be any nonnegative integer, $j$ and integer, $0 \leq j \leq k - 1$, and let

$$T(v, j, \chi) = \sum_{n=j+1}^{j+k} \frac{\chi(vk + n)}{vk + n} = \sum_{n=j+1}^{j+k} \frac{\chi(n)}{vk + n}.$$

Then $L(1, \chi) = \sum_{n=1}^{j} \frac{\chi(n)}{n} + \sum_{v=0}^{\infty} T(v, j, \chi)$.

In [3] Davenport proved the following theorem:

THEOREM (H. Davenport). *If $\chi(-1) = 1$, then $T(v, 0, \chi) > 0$ for all $v$ and $k$. If $\chi(-1) = -1$, then $T(0, 0, \chi) > 0$ for all $k$, and $T(v, 0, \chi) > 0$ if $v > v(k)$; but for any $r \geq 1$ there exist values of $k$ for which*

$$T(1, 0, \chi) < 0,\ T(2, 0, \chi) < 0, \ldots, T(r, 0, \chi) < 0.$$

In this paper, we will prove

THEOREM 2. *For fixed integers $k$ and $j$, $0 \leq j \leq k - 1$,*

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Received August 24, 1994.

This research was done when the first author was visiting the Pennsylvania State University, supported by a grant from the National Science Council of the Republic of China. He would like to thank the Mathematics Department of the Pennsylvania State University for its hospitality. The second author's research is supported in part by the NSA grants MDA904-92-H-3054 and MDA904-95-H-1006.