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A NOTE ON A FORMULA OF THE LÉVY-KHINCHIN TYPE IN QUANTUM PROBABILITY

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1. Introduction

In the past few years there has been an increasing interest in a certain class of stochastic differential equations (SDE's) in Hilbert spaces for applications in quantum mechanics (measurements continuous in time [1-5]) and in quantum optics (photon-detection theory and numerical simulations of *master equations* [6-10]). Part of the mathematical theory of these equations has been developed in [11], where also "structural properties" of this class of equations have been studied. In the paper [11] it has been shown that such equations are connected with certain semigroups of linear operators and the form of the generator of semigroups related to such SDE's has been established.

Independently of SDE's, semigroups of the same type have been studied in [12-14] and, under some mathematical restrictions, their generators have been completely classified through some quantum analogue of the Lévy-Khinchin formula. The problem is that the two forms of the generators obtained in the two papers [11, 14] are not directly comparable. The aim of the present note is to rewrite the generator obtained in [14] in an equivalent, more explicit form. At that point the expression for the generator of the semigroups considered in [11] will be comparable with the new form of the generator obtained in [14] and it will be evident that the semigroups studied in [11] are strictly a subclass of the ones of the article [14]. This will open the problem, at present under study, of constructing SDE's connected to semigroups with the same generality as those studied in the article [14]. This will give rise to new SDE's that haven't been taken into account in the physical literature up to now and that may have new physical applications.

Section two is dedicated to the statement of the problem and to the presentation of the results of the article [14]: semi-uniformly continuous semigroups of probability operator on von Neumann algebras and a quantum analogue of the

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