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SIEGEL MODULAR FORMS AND THETA SERIES ATTACHED TO QUATERNION ALGEBRAS II

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Abstract. We continue our study of Yoshida's lifting, which associates to a pair of automorphic forms on the adelic multiplicative group of a quaternion algebra a Siegel modular form of degree 2. We consider here the case that the automorphic forms on the quaternion algebra correspond to modular forms of arbitrary even weights and square free levels; in particular we obtain a construction of Siegel modular forms of weight 3 attached to a pair of elliptic modular forms of weights 2 and 4.

Introduction

We resume in this article our study of Yoshida's lifting from [5]. This lifting associates a Siegel modular form $Y^{(2)}(\varphi_1, \varphi_2)$ of degree 2 to a pair φ_1, φ_2 of automorphic forms on the adelic multiplicative group of a definite quaternion algebra D over $\mathbf{Q}$. This pair corresponds under Eichler's correspondence to a pair f, g of elliptic modular forms of weights $k_1 = 2$, k_2 and the same square free level N; it is a theta lifting from the orthogonal group of D equipped with the norm form to the group Sp_2. In the case that also $k_2 = 2$ holds we proved that this lifting is nonzero if both f, g are cusp forms with the same eigenvalues under the Atkin-Lehner involutions; if φ_2 is constant (and hence g is an Eisenstein series) the vanishing of $Y^{(2)}(\varphi_1, 1)$ depends on the central critical value of the L-function of f. We extended the construction to arbitrary pairs of weights k_1, k_2 in [6], where we also gave the analogous construction of a lifting $Y^{(n)}(\varphi_1, \varphi_2)$ to Siegel modular forms of higher degree. These higher degree liftings are in general vector valued Siegel modular forms; $Y^{(n)}(\varphi_1, \varphi_2)$ is mapped to $Y^{(n-1)}(\varphi_1, \varphi_2)$ by Siegel's ϕ-operator.

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