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A GENERALIZATION OF KITA AND NOUMI'S VANISHING THEOREMS OF COHOMOLOGY GROUPS OF LOCAL SYSTEM

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Abstract. We prove vanishing theorems of cohomology groups of local system, which generalize Kita and Noumi's result and partially Aomoto's result. Main ingredients of our proof are the Hodge to de Rham spectral sequence and Serre's vanishing theorem in algebraic geometry.

§1. Introduction

Vanishing theorems of cohomology groups of local system are important to study the theory of hypergeometric functions (cf. [AK]). In fact, several vanishing theorems of cohomology groups of local system are known. For example, Kita and Noumi [KN] obtained a vanishing theorem by using the technique of a filtration attached to logarithmic forms. Our aim is to generalize their result and give a simple algebro-geometric proof. Of course there is a tradeoff: While their proof is concrete and elementary, ours is abstract and depends on big theorems.

In order to state our theorem, we begin with the definition on the sheaf of logarithmic 1-forms along a divisor adapted in this paper, which is different from the one given in [S]. Let M be a complex projective manifold of dimension n, D an effective reduced divisor on M. Let $D = \sum_{j=1}^{m} D_j$ be the irreducible decomposition of D and f_j the defining equation of D_j. For $x \in M$, Assume $x \in D_{j_i}, i = 1, \dots, k$ and $x \notin D_j$ for $j \neq \{1, \dots, m\} \setminus \{j_1, \dots, j_k\}$. Then the sheaf $\Omega^1(\log D)$ of logarithmic 1-forms along D is defined as follows: $\psi \in \Omega^1(\log D)_x$ if and only if $\psi = \sum_{i=1}^{k} h_k \frac{df_{j_i}}{f_{j_i}} + \varphi$, where h_i is a holomorphic function at x and φ is a holomorphic 1-form at x. We also define $\Omega^p(\log D)$ as $\wedge^p \Omega^1(\log D)$. Let $\mathcal{E}$ be a locally free sheaf on M. A meromorphic connection ∇ on $\mathcal{E}$ is said to be an integrable holomorphic

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