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TRANSFORMS ON WHITE NOISE FUNCTIONALS WITH THEIR APPLICATIONS TO CAUCHY PROBLEMS

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Abstract. A generalized Laplacian $\Delta_G(K)$ is defined as a continuous linear operator acting on the space of test white noise functionals. Operator-parameter $\mathcal{G}_{A,B}$- and $\mathcal{F}_{A,B}$-transforms on white noise functionals are introduced and then prove a characterization theorem for $\mathcal{G}_{A,B}$ and $\mathcal{F}_{A,B}$-transforms in terms of the coordinate differential operator and the coordinate multiplication. As an application, we investigate the existence and uniqueness of solution of the Cauchy problem for the heat equation associated with $\Delta_G(K)$.

§1. Introduction

Gross [5] initiated the study of the theory of differential equations on infinite dimensional abstract Wiener space (H, B). Suppose φ is a twice H-differentiable function on B such that its second H-derivative φ'' is a trace class operator of H. Then the Gross Laplacian $\Delta_G\varphi$ of φ is defined by

$$\Delta_G\varphi = \mathrm{trace}_H\, \varphi''.$$

In [5] Gross studied the solution of the Cauchy problem for the heat equation associated with the Gross Laplacian Δ_G:

$$\frac{\partial u(x,\theta)}{\partial \theta} = \frac{1}{2}\Delta_G u(x,\theta).$$

It has been shown [5] that the solution can be represented as an integral with respect to the abstract Wiener measure. For further works see [10], [15], [19].

Based on the white noise analysis, Kuo [11] formulated the Gross Laplacian Δ_G and the number operator N in terms of the Hida differentiation ∂_t

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