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Nagoya Math. J.
Vol. 149 (1998), 117–171

ON TWISTING OPERATORS AND NEWFORMS OF HALF-INTEGRAL WEIGHT II

COMPLETE THEORY OF NEWFORMS FOR KOHNEN SPACE

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Abstract. The author continues his previous work with the purpose of establishing a theory of newforms in the case of half-integral weight. In this paper, the author formulates and proves a complete theory of newforms for Kohnen space. Kohnen space is an important canonical subspace in the space of cusp forms of half-integral weight $k + 1/2$ ($k > 0$). Every Hecke common eigenform in Kohnen space corresponds to a primitive form of integral weight $2k$ and of odd level via Shimura Correspondence.

These newforms for Kohnen space satisfy the Strong multiplicity One theorem. Moreover, we explicitly determine the corresponding primitive form (of weight $2k$) to each newform for Kohnen space. The space of oldforms is also explicitly described.

In order to find all newforms for Kohnen space, the author needs a certain non-vanishing property of Fourier coefficients of cusp forms. Such property proves by using representation theory of finite linear groups. The method of proof of newform theory is mainly based on trace formulae and trace relations.

Introduction

The theory of newforms is very important and useful for arithmetical study of modular forms of integral weight. This theory has the following nice properties:

(i) The space of newforms have an orthogonal $\mathbf{C}$-basis consisting of common eigenforms for all Hecke operators and such common eigenforms are uniquely determined up to multiplication of complex numbers. Moreover such common eigenforms satisfy *the Strong Multiplicity One Theorem* (cf. [M, §4.6]).

Received June 13, 1996.
[1]Partly supported by the Grants-in Aid for Scientific Research, the Ministry of Education of Japan.