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ON THE DETERMINISM OF THE DISTRIBUTIONS OF MULTIPLE MARKOV NON-GAUSSIAN SYMMETRIC STABLE PROCESSES

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Abstract. Consider a non-Gaussian $S\alpha S$ process $X = \{X(t); t \in T\}$ which is expressed as a canonical representation $X(t) = \int_{u \leq t, u \in T} F(t,u)\, dZ(u)$, $t \in T$, and is continuous in probability. If X is n-ple Markov, then X has determinism of dimension $n+1$. That is, any $S\alpha S$ process $\tilde{X} = \{\tilde{X}(t); t \in T\}$ having the same $(n+1)$-dimensional distributions with X is identical in law with X.

§1. Introdution

In this paper we consider the determinism of the distribution of an $S\alpha S$ (= symmetric α-stable) random field $(0 < \alpha \leq 2)$ in the following sense.

DEFINITION. We say that an $S\alpha S$ random field $X = \{X(s); s \in S\}$ has *determinism of dimension n* if any $S\alpha S$ random field $\tilde{X} = \{\tilde{X}(s); s \in S\}$ having the same n-dimensional distributions with X is identical in law with X.

In this definition, "X and $\tilde{X}$ have the same n-dimensional distributions" means that $(X(s_1), X(s_2), \cdots, X(s_n))$ and $(\tilde{X}(s_1), \tilde{X}(s_2), \cdots, \tilde{X}(s_n))$ have a common distribution for any choice of distinct $s_1, s_2, \cdots, s_n \in S$. "$X$ and $\tilde{X}$ are identical in law" means that they have the same finite-dimensional distributions of all dimensions. Obviously, if X has determinism of dimension n, then X has determinism of dimension m for $m > n$. A centered Gaussian random field is symmetric stable with the index $\alpha = 2$ and has determinism of dimension 2 because any finite-dimensional distribution is expressed by its covariance function. On the other hand, it is not easy to answer a question whether a particular non-Gaussian $S\alpha S$ random field $(0 < \alpha < 2)$ has determinism of a given dimension or not. However, the determinism of some non-Gaussian $S\alpha S$ random fields has been studied.

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