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EXTENSION OF CR STRUCTURES ON THREE DIMENSIONAL PSEUDOCONVEX CR MANIFOLDS

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Abstract. Let $\overline{M}$ be a smoothly bounded orientable pseudoconvex CR manifold of finite type and $dim_{\mathbb{R}} M = 3$. Then we extend the given CR structure on M to an integrable almost complex structure on S_g^+ which is the concave side of M and $M \subset bS_g^+$.

§1. Introduction

Let $\widetilde{M}$ be a smooth orientable manifold of dimension $2n-1$ and let $\overline{M} \subset \widetilde{M}$ be a smoothly bounded CR manifold with a given CR structure $\mathcal{S}$ of dimension $n-1$. Since $\widetilde{M}$ is orientable, there are smooth real nonvanishing 1-form η and smooth real vector field X_0 on $\widetilde{M}$ so that $\eta(X) = 0$ for all $X \in \mathcal{S}$ and $\eta(X_0) = 1$. We define the Levi form of $\mathcal{S}$ on $\overline{M}$ by $i\eta([X', \overline{X}''])$.

In [4], Catlin has considered an extension problem of a given CR structure on M to an integrable almost complex structure on a $2n$-dimensional manifold Ω with boundary so that the extension is smooth up to the boundary and so M lies in $b\Omega$. Under certain conditions on the Levi form (cf., [4, Theorem 1.1, Theorem 1.3]), this leads to a solution of the Kuranishi problem [1, 9, 13], which is to show that an abstract CR manifold can be locally embedded in $\mathbb{C}^n$.

In this paper, we consider an extension problem of a given CR structure on M when M is a pseudoconvex CR manifold of finite type and $\dim_{\mathbb{R}} M = 3$. For a given positive continuous function g on M, where $g = 0$ on bM, we define

$$S_g^+ = \{(x,t) \in M \times [0,\infty);\ 0 \le t \le g(x)\}.$$

Then our main result is the following theorem:

THEOREM 1.1. *Let $\overline{M} \subset \widetilde{M}$ be a smoothly bounded orientable pseudoconvex CR manifold of finite type with given CR structure $\mathcal{S}$ on M and*

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