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LIMIT THEOREMS FOR HITTING TIMES OF 1-DIMENSIONAL GENERALIZED DIFFUSIONS

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Abstract. Limit theorems are obtained for suitably normalized hitting times of single points for 1-dimensional generalized diffusion processes as the hitting points tend to boundaries under an assumption which is slightly stronger than that the existence of limits $\gamma + 1$ of the ratio of the mean and the variance of the hitting time. Laplace transforms of limit distributions are modifications of Bessel functions. Results are classified by the one parameter $\{\gamma\}$, each of which is the degree of corresponding Bessel function. In case the limit distribution is degenerate to one point, by changing the normalization, we obtain convergence to the normal distribution. Regarding the starting point as a time parameter, we obtain convergence in finite dimensional distributions to self-similar processes with independent increments under slightly stronger assumption.

§1. Introduction

We denote by $\mathcal{M}$ the class of right continuous non-decreasing functions $m : [-\infty, \infty] \to [-\infty, \infty]$, satisfying $m(-\infty) = -\infty$, $m(\infty) = \infty$, $m(0-) = 0$. For $m \in \mathcal{M}$, we set

$$l_1(m) = \sup\{x < 0 \ ; \ m(x) = -\infty\},$$
$$l_2(m) = \inf\{x > 0 \ ; \ m(x) = \infty\}.$$

If there is no confusion, we write $l_i(m)$ simply l_i for $i = 1, 2$. We denote by E_m the support of the measure induced by m restricted to (l_1, l_2). There naturally corresponds a strong Markov process $\{X_t\}$ (called 1-dimensional generalized diffusion process) on E_m (whose formal infinitesimal generator is $\frac{d}{dm}\frac{d}{dx}$) to m by changing time of the Brownian motion. The measure $m(dx)$ is called the speed measure of $\{X_t\}$. Denote the hitting time of x for $\{X_t\}$ by τ_x. We are concerned with a problem what is the suitable normalization and what is the limit distribution of τ_x when the process starts at the origin and x tends to l_2.

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